Handout: Turbulent Mixing Layer

- Flow between two uniform parallel streams of different velocities $U_h > U_l \geq 0$

  $U(y<0,x=0)=U_l$, $U(y>0,x=0)=U_h$

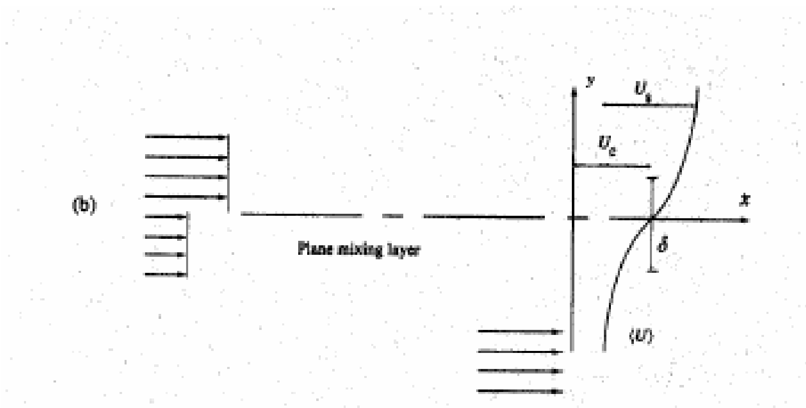

Fig. 5.14 from Pope's Turbulent Flows book

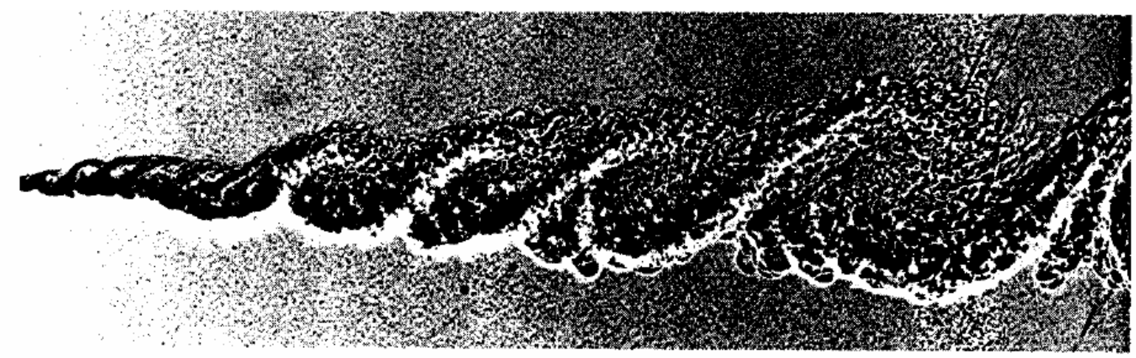

Fig. 5.51. A visualization of the flow of a plane mixing layer. A spark shadow graph of a mixing layer between helium (upper) $U_h = 10.1$ m s^{-1} and nitrogen (lower) $U_l = 3.8$ m s^{-1} at a pressure of 8 atm. (From Brown and Roshko (1974).)

- Two velocities $\rightarrow$ new parameter U_l / U_h

$$\frac{\langle U(x,t)\rangle}{U_h} = f(\mathrm{Re}, \frac{y}{x}, \frac{U_l}{U_h})$$

where <> denotes a mean (Reynolds averaged) quantity

$\rightarrow$ Mixing layer is self-similar

- Characteristic velocity

$$U_c = \frac{1}{2}(U_h + U_l)$$

- Velocity difference

$$U_s = U_h - U_l$$

- Characteristic mixing layer width

With $y_\alpha(x)$ defined by $\langle U(x, y_\alpha(x)) \rangle = U_l + \alpha U_s$

$$\delta(x) = y_{0.9}(x) - y_{0.1}(x)$$

- Reference lateral position

$$\bar{y}(x) = \frac{1}{2}(y_{0.9}(x) + y_{0.1}(x))$$

- Scaled cross-stream coordinate

$$\xi = \frac{y - \bar{y}(x)}{\delta(x)}$$

- Scaled velocity

$$f(\xi) = \frac{< U > - U_c}{U_s}$$

$$f(\pm\infty) = \pm\frac{1}{2}$$

$$f(\pm\frac{1}{2}) = \pm 0.4$$

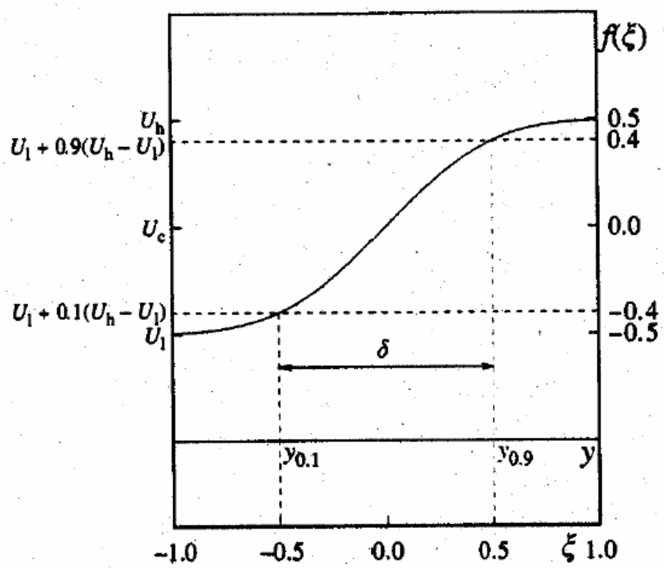

Fig. 5.21. A sketch of the mean velocity $\langle U \rangle$ against y, and of the scaled mean velocity profile $f(\xi)$, showing the definitions of $y_{0.1}$, $y_{0.9}$, and δ.

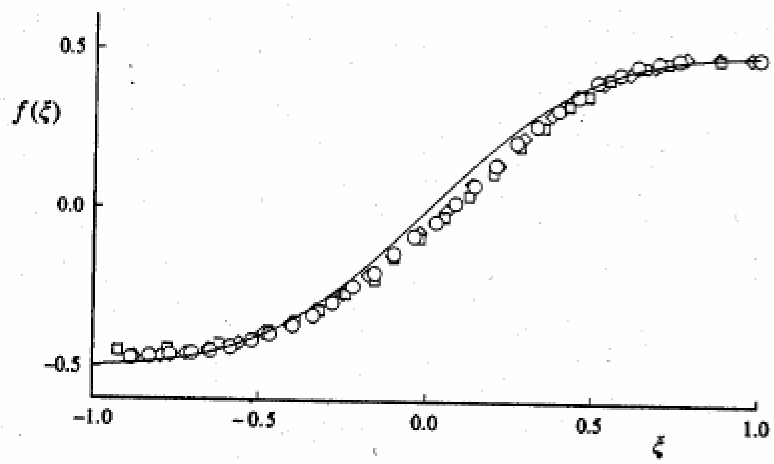

Fig. 5.22. Scaled velocity profiles in a plane mixing layer. Symbols, experimental data of Champagne *et al.* (1976) (○, x = 39.5 cm; □, x = 49.5 cm; ○, x = 59.5 cm); line, error-function profile (Eq. (5.224)) shown for reference.

- Flow not symmetric around y=0

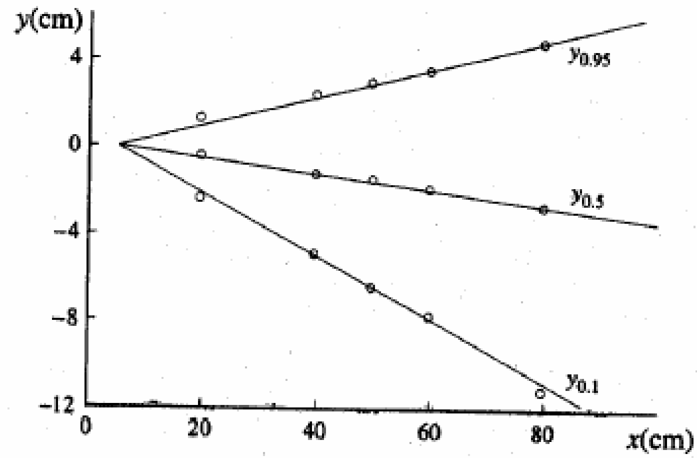

Fig. 5.23. Axial variations of $y_{0.1}$, $y_{0.5}$, and $y_{0.95}$ in the plane mixing layer, showing the linear spreading. Experimental data of Champagne *et al.* (1976).

Spreading Rate

$$\frac{d\delta(x)}{dx} = const = \frac{U_s}{U_c} S$$

$$\Rightarrow S = \frac{U_c}{U_s}\frac{d\delta(x)}{dx} \quad S \approx 0.06 - 0.11 \text{ is independent of } \frac{U_s}{U_c}$$

The variation in the range of recorded values depends on the state of the flow as it leaves the splitter plate.

Turbulent Kinetic Energy Flow Rate

$$K(x) \sim \int_{-\infty}^{\infty} < U > k dy \sim U_c U_s^2 \delta \sim x \quad \text{as } \delta \sim x$$

$K(x)$ increasing with x

$$\Rightarrow P > \varepsilon \quad (\textit{Rogers} \ \& \ \textit{Moser} \ (1994): \frac{P}{\varepsilon} = 1.4)$$

Statistics for self-similar plane mixing layer

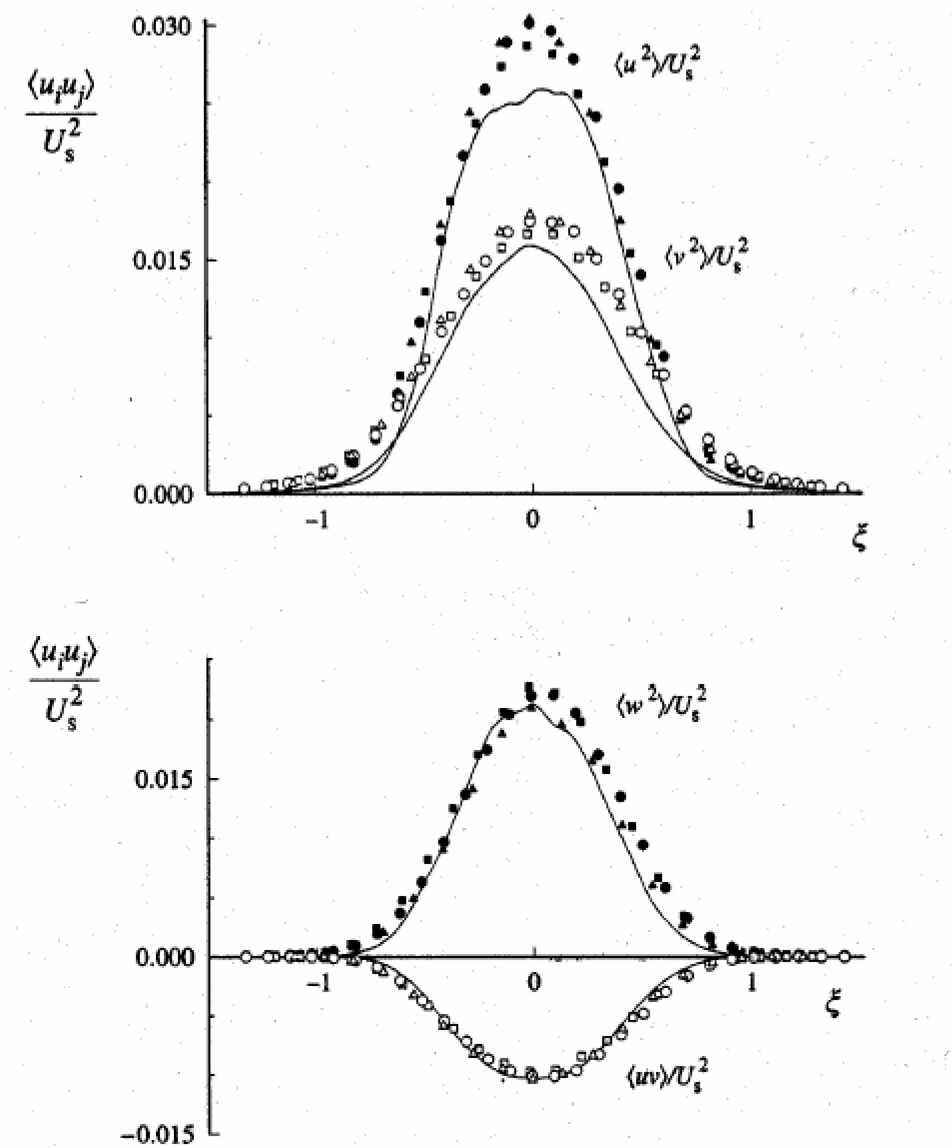

Fig. 5.25. Scaled Reynolds-stress profiles in self-similar plane mixing layers. Symbols, experiment of Bell and Mehta (1990) ($U_l/U_h = 0.6$); solid line, DNS data for the temporal mixing layer (Rogers and Moser 1994).

Temporal Mixing Layer

- Limit:

$$\frac{U_s}{U_c} \to 0 \text{ or } \frac{U_l}{U_h} \to 0$$

- Boundary layer equation for self-similar mixing layer

(g=<uv>/U_s^2, f'=df/dξ, g'=dg/dξ)

$$\left(\frac{U_c}{U_s}\frac{d\delta}{dx}\right)\left(\xi+\frac{U_s}{U_c}\int_0^{\xi} f(\xi')d\xi'\right)f'=g'$$

reduces to

$$U_c\,\frac{\partial\langle U\rangle}{\partial x}=-\frac{\partial\langle uv\rangle}{\partial y}$$

with $\tau = x/U_c$

$$\frac{\partial\langle U\rangle}{\partial \tau}=-\frac{\partial\langle uv\rangle}{\partial y}$$

An observer traveling in the x direction at speed U_c sees two streams moving to right and left with velocities $0.5U_s$ and $-0.5U_s$ at $\pm\infty$ Gradients of mean quantities in the x direction are vanishingly small (of order of U_s/U_c) compared with gradients in y direction. The thickness of the mixing layer grows it time at rate SU_s. Thus, in the moving frame, as U_s/U_c tends to zero, the flow becomes statistically one-dimensional and time-dependent. It is called the temporal mixing layer and it is statistically symmetric about y=0.