Handout: Large Eddy Simulation IV

Wall Models & Hybrid RANS-LES methods

Wall models in LES

- At high Re, full-resolved LES is prohibitively expensive due to the small but dynamically important eddies in the near-wall region
- Wall modeling required for LES at high Reynolds numbers especially in the attached boundary layers
- Classical solution is to use coarse near-wall grid and supply wall stresses in cells adjacent to wall
- Hybrid LES/RANS methods are also possible where the RANS model is used as a 'near wall' model in LES

Why full resolved LES is too expensive?

Problem with LES of wall-bounded flows (e.g., channel flow):

- Near wall region has small structures (streaks).

- Their dimensions in wall units are:

$L^+ = 1000; W^+ = 20; H^+ = 30$

- Much finer mesh has to be employed compared to the grid in outer region to resolve these structures

- Required spacing: $\Delta x^+ = 100; \Delta y^+ = 1; \Delta z^+ = 5$

- To resolve the near-wall eddies, the number of grid points required is proportional to Re^2, nearly as many as for DNS

- Cost enormous at high Re

- Result: cannot simulate complex flows at realistic Reynolds numbers

Additional problems for separated flows:

- At separation, boundary layer becomes a free shear layer
- At reattachment, opposite occurs
- Griding and modeling are difficult in terms of tracking the attached boundary layers
- Two types of separation are possible:

- **Abrupt**: usually caused by geometry (step) or discontinuity (shock)
  - Example: backward facing step
- **Slow**: boundary layer is on the verge of separation ($C_f \approx 0$) for a long distance
  - Occurs in diffusers, airfoils
  - Second type much harder to simulate

Do we have to always resolve the near wall region?

Are the near wall and outer regions strongly coupled?

a) Near-wall region simulation (Kuhn & Chapman,1985)

- Used boundary condition to represent outer region
- Had fluctuations introduced at correct time and length scales
- Top boundary was situated at $y^+ = 100$
- Results are good representation of near-wall flow
- Had streaks and other structures, correct fluctuations
- Only case done had zero-pressure gradient

b) Outer region simulation

- Use boundary condition to represent wall region
- This requires use of a wall model at the bottom boundary
- Typically assumes logarithmic law of the wall valid in order to calculate wall stress
- First point off the wall needs to be situated in the log layer
- Position depends on Reynolds number, at high Re, $y^+ \approx 100\text{-}500$

- Similar to 'law of wall' condition in RANS
- Need to include fluctuations

Simulations mentioned above show:

- Inner and outer layers may not be strongly coupled
- Can compute either one without the other
- In most cases we want to simulate using LES only outer region (less expensive)
- If only outer layer simulated, the saving are enormous at very high Re

  - Can use much coarser grid

  - Not much need for inner layer simulation if the wall model supplies the correct instantaneous wall shear stresses

'Simple' wall stress models in LES

- Simple wall stress models are analogous to the wall functions commonly used in RANS approaches except that they are applied in the instantaneous sense in time-accurate calculations.

- RANS wall-functions models work well in 'equilibrium' flow. Not successful in transition, separated flow

- Mean flow must have logarithmic behavior at lowest simulated level for the model to be successful

- All 'simple' wall-stress models imply the logarithmic (or power) law of the wall for the mean velocity, which is not valid in many complex flows, especially if separation is present.

- Additional modeling required if surface is rough

Requirements of a good 'simple' LES wall stress model

- Must produce log profile near lower boundary of domain

- Should handle satisfactory:

  - Pressure gradients (adverse and favorable)
  - Transpiration (blowing and suction)
  - Separation and reattachment

- Very challenging for existing models, not really possible right now

Purpose of wall model in LES

- Acts as boundary condition for simulation
- The wall functions provide an algebraic relationship between the local wall stresses and the tangential velocities at the first point off the wall.
- Should include shear stress at wall in condition
- In the wall normal direction for an impermeable surface the normal velocity component is set to zero.

Accuracy required depends on application:

- Meteorology≈20% OK
- Engineering ≈1% sometimes needed (airplane drag coefficient)
- Good model in one area not necessarily good model for other areas

'Simple' wall models in LES (brief description)

1) Schumann model (1976)

- Was developed specifically for channel flow calculations
- Assumes linear relation between instantaneous (resolved) streamwise velocity at first grid point off the wall and instantaneous wall shear stress.

$$\tau_{12}(x,z) = \frac{\overline{u}_1(x,y_1,z)}{U_1(y_1)}\langle \tau_w \rangle$$

where the wall normal direction is y, y_1 corresponds to the first point off the wall and <> represents **time** average.

- Values of mean streamwise velocity at first point off the wall $U_1(y_1) = \langle \overline{u}_1(x,y_1,z) \rangle$ provided or computed. For instance one can get $U_1(y_1)$ from the logarithmic law

- Skin friction $\langle \tau_w \rangle$ should be provided (for a plane channel flow it is equal to the driving mean pressure gradient; for more complex flows one can get it from a separate RANS simulation)
- Normal boundary condition $\bar{u}_2(y=0)=0$ (impermeability).
- If there is no mean flow in the other direction parallel to the wall (z), the instantaneous wall shear stress $\tau_{32}(x,z)$ is determined from

$$\tau_{32}(x,z)=\frac{\bar{u}_3(x+\Delta_s, y_1, z)}{U_1(y_1)}\langle \tau_w \rangle$$

This is equivalent to assuming

$$\frac{\tau_{12}(x,z)}{\tau_{32}(x,z)} \approx \frac{\bar{u}_1(x,y_1,z)}{\bar{u}_3(x,y_1,z)}=\frac{\nu_t(\bar{u}_1(x,y_1,z)-0)/y_1}{\nu_t(\bar{u}_3(x,y_1,z)-0)/y_1}$$

i.e., a linear velocity profile and a constant eddy viscosity in the first grid cell off the wall.

2) Grotzbach Model (1987)

- Extends Shumann's model to avoid having to know the mean wall shear stress a priori.

- The average operator <> corresponds now to a mean over the plane parallel to the solid wall located at $y=y_1$ (flow is homogeneous in planes parallel to the walls).

- Once we calculate $U_1(y_1)=<\bar{u}_1(x,y_1,z)>$ we can estimate the friction velocity u_τ and then the mean wall shear stress $\langle \tau_w \rangle = \rho u_\tau^2$ from the logarithmic law:

$$u_1^+(y_1)=<\bar{u}_1(x,y_1,z)>/u_\tau=\frac{1}{\kappa}\log\left(y_1 u_\tau/\nu\right)+B$$

- More flexible but still relays on the logarithmic law.

c) Shifted Correlations Model (Piomelli et al., 1989)

- Similar to Schumann' model but incorporates information on flow structure in wall region
- Inclined coherent structures in near wall region are responsible for the velocity fluctuations and wall shear stress. The average value of this angle is about 8^0 very close to the wall and increases slowly away from the wall based on the experimental work of Rajagopalan and Antonia.
- The model requires the wall stress to be correlated to the instantaneous velocity some distance Δ_s downstream of the point the wall stress is required:

$$\tau_{12}(x,z) = \frac{\bar{u}_1(x+\Delta_s, y_1, z)}{U_1(y_1)} \langle \tau_w \rangle$$

$$\bar{u}_2(y=0) = 0$$

$$\tau_{32}(x,z) = \frac{\bar{u}_3(x+\Delta_s, y_1, z)}{U_1(y_1)} \langle \tau_w \rangle$$

- Δ_s is chosen to correspond to an angle of 8-13 ° :

$$\Delta_s = y_1 \cot(8^0) \text{ if } 30 < y_1^+ < 60$$

$$\Delta_s = y_1 \cot(13^0) \text{ if } y_1^+ > 60$$

- Correct value for zero pressure gradient, not known for others

d) Ejection Model (Piomelli et al., 1989)

- similar to Grotzbach's model
- takes into account the effect of sweep and ejection events on the wall shear stress
- The impact of fast fluid pockets on the wall causes the longitudinal and lateral vortex lines to stretch out, increasing velocity fluctuations near the wall. The

ejection of fast fluid masses induces the inverse effect, i.e. reduces the wall shear stress

- Use normal velocity $\bar{u}_2$ in model, include the shift of the ejection model

$$\tau_{12}(x,z) = \langle \tau_w \rangle - C u_\tau \bar{u}_2 (x + \Delta_s, y_1, z)$$

$$\bar{u}_2 (y = 0) = 0$$

$$\tau_{32}(x,z) = \langle \tau_w \rangle / U_1(y_1) \bar{u}_3 (x + \Delta_s, y_1, z)$$

- Δ_s as above, C is a constant of order of unity
- Again, only valid for zero pressure gradient and assumes logarithmic law to calculate mean wall shear stress

e) Werner-Wengle Model

- Variant of Grotzbach's model
- Based on power law (1/7) profile for the streamwise velocity instead of the logarithmic law
- Assumptions:
    - instantaneous velocity components at the wall in the directions parallel to the wall are in phase with the associated wall shear stresses
    - instantaneous velocity profile given by

$$u^+(y) = y^+ \quad if \quad y^+ < 11.8$$

$$u^+(y) = 8.3 (y^+)^{1/7} \quad if \quad y^+ > 11.8$$

The values of the tangential velocity components can be related to the corresponding values of the wall shear stress components by integrating the velocity profile given above over the distance separating the first cell from the wall.

- Main advantage: One can analytically evaluate the wall shear stress components from the velocity field
- Fairly widely used

<u>f) Mason and Callen Model</u>

- Specifically designed for rough walls

$$\bar{u}_1(x, y_1, z) = \cos\theta\left(\frac{u_\tau(x,z)}{\kappa}\right)\ln(1 + y_1 / y_0)$$

$$\bar{u}_2(y = 0) = 0$$

$$\bar{u}_3(x, y_1, z) = \sin\theta\left(\frac{u_\tau(x,z)}{\kappa}\right)\ln(1 + y_1 / y_0)$$

$$\theta = \arctan(\bar{u}_3(x, y_1, z) / \bar{u}_1(x, y_1, z))$$

where y_0 is the roughness thickness

- The squared modulus of the instantaneous surface friction vector is then:

$$u_\tau^2 = \frac{1}{M}\sqrt{\bar{u}_1^2(x, y_1, x) + \bar{u}_3^2(x, y_1, x)}$$

$$\frac{1}{M} = \frac{1}{\kappa^2}\ln^2(1 + y_1 / y_0)$$

- The model is based on the hypothesis that the logarithmic velocity distribution is verified locally and instantaneously by the velocity field
- Popular in meteorology, not engineering

Experience with these ‘simple’ LES wall stress models:

- Work well in attached flow
- Including transpiration, pressure gradient , etc.
- Reduce computation time by factor of 10 or more
- Separated flows
  - Not predicted accurately

- Worse at low Reynolds numbers, better at higher Re
- Not clear whether good model exists for all conditions

- More accurate results using these models for complex geometries can be obtained if the distribution of the mean wall shear stress is known. If the mean wall stress is obtained from a precalculated RANS solution of the same flow, than models based on Schumann's approach can be used quite successfully.
- Additionally, if the flow is statistically 2D the best way to determine the mean velocity needed at the first point off the wall is to perform spanwise averages at each time step during simulation. Such an approach was used by Wu and Squires (1998) to perform LES of a 3D boundary layer over a swept bump. The dynamic Smagorinsky model was used as the base LES model.

More complex models:

- Use RANS model for inner part of boundary layer (original idea by Balaras et al., 1996 and Cabot, 1996)
- Couple to LES for region above the inner part of boundary layer
- Compute both parts simultaneously
- Can handle pressure gradient, transpiration, etc.
- Latest model: dynamic wall model of Wang and Moin, 2002

Dynamic wall model of Wang and Moin (2002)

- The main idea is to use RANS near wall, but to adjust RANS coefficients dynamically to match LES at the boundary between RANS and LES.
- The **RANS model is based on turbulent boundary layer (TBL) equations**; simpler variants of the full equations can employed to compute the instantaneous wall shear stress, which is used as approximate boundary condition for LES.
- The values of the eddy viscosity in the wall layer are reduced compared to the typical RANS values to account only for the unresolved part of the Reynolds stress in the wall layer. This is done using a **dynamically adjusted mixing-length eddy viscosity** in the TBL equation model.

- This procedure was shown to be considerably more accurate than the simpler wall models described above
- **The TBL equations are solved numerically on an embedded near-wall mesh to compute the wall stress**
- The TBL equations are forced at the outer layer (boundary between RANS and LES region) by the instantaneous tangential velocities from LES, while no-slip conditions for the velocity are applied at the wall.
- The **turbulent viscosity is modeled** in the original version of the model **with a mixing-length model with wall damping**, but the **model can be refined to incorporate more advanced RANS closures** that use transport equations for the turbulent quantities.

$$\frac{\nu_t}{\nu} = \kappa y_w^+ (1 - e^{-y_w^+/A})^2 \qquad (1)$$

where $y_w^+ = y_w u_\tau / \nu$ is the distance to the wall in wall units, κ is the model coefficient (in RANS κ corresponds to the von Karman constant that is equal to 0.4), and A=19. The pressure in equations (3) and (4) is assumed not to vary significantly in the wall normal direction and is equal to the value from the outer flow LES solution (first point off the wall in the LES grid). Equations (3) are required to satisfy the no-slip conditions on the wall and match the outer layer solutions at the first off-wall LES velocity nodes:

$u_i = 0$ at the wall

and (2)

$u_i = u_{i,LES}$ at $x_2=\delta$ (i=1 and i=3)

where δ is the local thickness of the wall layer grid.

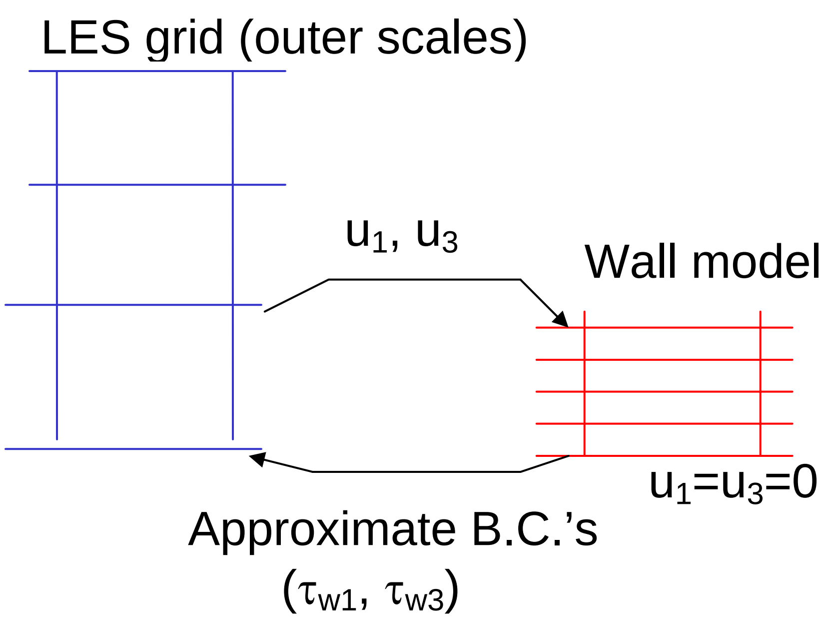
In general, the full TBL equations (3) are solved numerically to obtain u_1 and u_3. The boundary layer equations are integrated in time along with the outer flow LES equations. The wall-normal velocity component in the wall layer region is determined from the divergence free constraint (continuity). Unlike in the LES region, no Poisson equation is required to be solved since the pressure is assumed constant in the wall-normal direction in the inner region.

The wall-normal velocity ($u_2 = 0$) component is set to zero at the wall. For the tangential velocities the boundary conditions are imposed in terms of the shear stress τ_{w1}, τ_{w3} from wall models of the form:

$$\frac{\partial}{\partial x_2}(\nu + \nu_t)\frac{\partial u_i}{\partial x_2} = F_i \quad \text{i=1 or i=3} \tag{3}$$

where

$$F_i = \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} \tag{4}$$



The friction velocity

$$u_\tau = \left((\tau_{w1} / \rho)^2 + (\tau_{w3} / \rho)^2 \right)^{1/2} \qquad (5)$$

can be calculated using the instantaneous stresses from the previous time step, or using the stresses at the current time step (fully coupled) and an iterative procedure.

Since the grid in the directions parallel to the wall is the same in both the LES and TBL regions, and because the velocities are matched at the boundary between these regions, the resolved portions of the nonlinear stresses are approximately the same. This means that the TBL region contains turbulent structures too, so **the RANS model should account only for the unresolved part of the total turbulent stress**. Thus, the eddy viscosity predicted by the model used in the wall layer should be lower than the values predicted in full RANS simulations. To do that in the present model the constant κ is allowed to fluctuate such that at the boundary between the two regions **the mixing-length eddy viscosity and the SGS viscosity (given by LES) are equal**. From equation (1) we require:

$$\kappa = < \nu_{SGS} > / < \nu y_w^+ (1 - e^{-y_w^+ / A})^2 > \qquad (6)$$

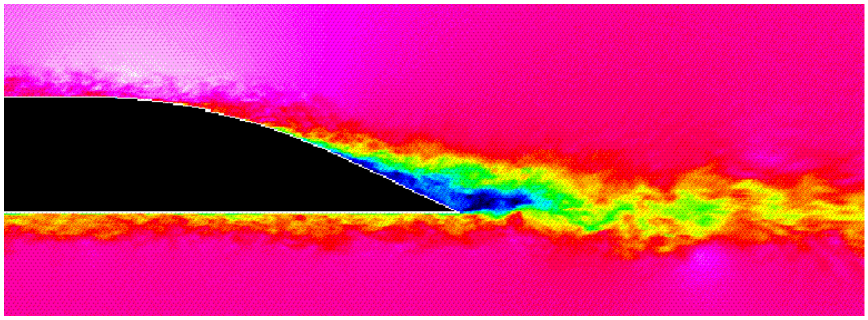
where the brackets <> denote averaging in time (~100 time steps back in time). The averaging is recommended to reduce the point to point oscillations in the model constant κ. For attached turbulent boundary layers the value of κ was estimated with the present model at around 0.1 which is significantly lower than von Karman's constant value.

The grid spacing in the TBL region in the wall-parallel directions is similar to the LES grid. In the normal direction a fine mesh ($\Delta y^+ < 3$) with the first point off the wall at approximately $\Delta y^{+} = 1$ is recommended. The first point off the wall in the LES grid is situated at 30-200 wall units.

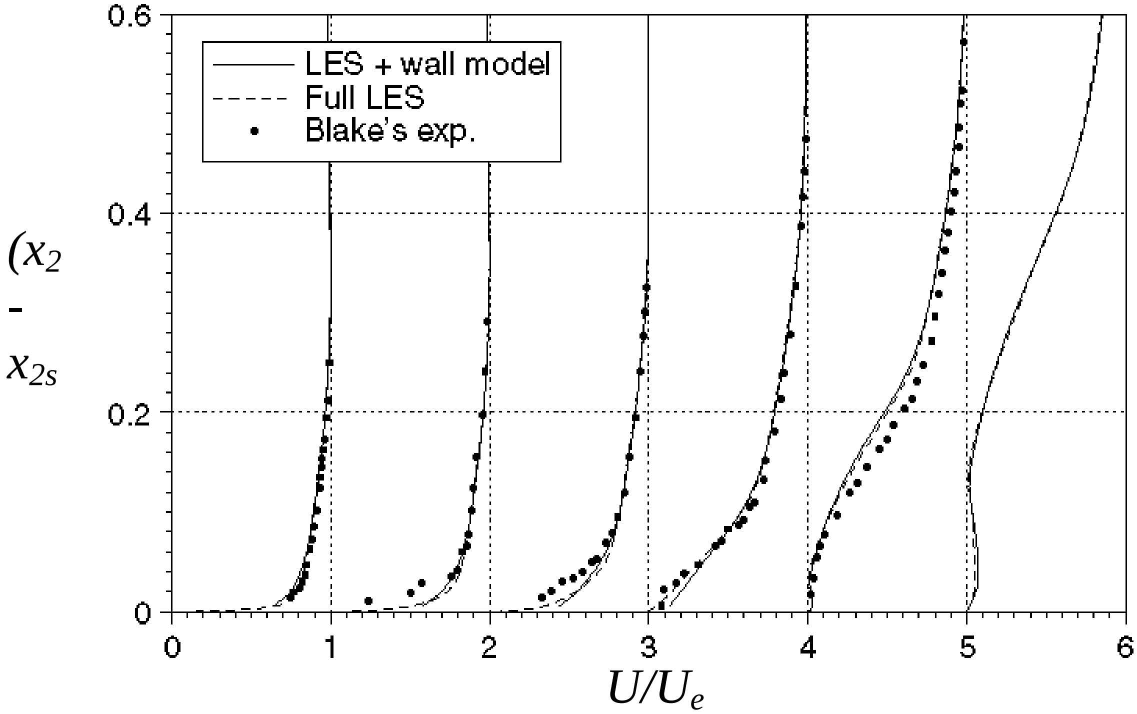
There are several simpler variants of the model described above obtained by retaining only the pressure term contribution in the definition of F_i in equation (3), or simply by setting **F_i=0**, in which case the new model is called the **equilibrium stress balance model**. In this simpler case, one can show that the algebraic model implies the logarithmic law of the wall for the instantaneous velocities for $\delta^{+} >> 1$ and linear velocity distribution for $\delta^{+} << 1$. Moreover equation (2) can be integrated numerically directly between the wall and the first point off the wall in the LES grid, without need to build a wall-layer grid which greatly simplifies the implementation of the wall-layer model in complex geometries.

The computational cost to solve the TBL equations is insignificant compared to the outer layer LES as there is no need to solve the momentum equation in the wall normal direction and the Poisson equation for the pressure. Also the TBL equations are much simplified in the locally orthogonal wall-layer coordinates (no cross-derivative terms).

The method was shown to predict low-order statistics (e.g., mean velocities) in good agreement with those from full (well-resolved) LES with resolved wall layers, at a much smaller computational cost (~one order of magnitude lower compared to the well resolved calculation) for several complex turbulent flows (e.g., swept airflow).



Flow over a swept airfoil (from Wang and Moin, 2002)



Comparison between full LES, LES with the dynamic wall model and experiment (from Wang and Moin, 2002)

Canopy models

- Designed originally for meteorological boundary layer applications
- **Surface is always rough**
- Represent near-wall region as a porous medium (e.g., see Brown et al., 2001, Cederwall and Street, 2001).
- **Main idea**: Employ a 'canopy' layer to represent the additional SGS turbulent transfer that has been missing near the surface.
- The rationale of the canopy model has also an interpretation in the representation of the flow on a grid:
  * Far from the wall, the filtered flow represents most of the flow energy with the SGS terms carrying the energy held on the grid beyond the resolved scales.
  * As the wall is approached two things happen:
    - More and more energy is neither resolved nor held on the grid and so it is truly 'subgrid' and not represented
    - Due to the typical grid anisotropy $\Delta z << \Delta x \quad and \quad \Delta y$ (z direction normal to the canopy plane); thus near the wall eddies smaller than Δx in diameter are not properly represented. The task of the canopy model is to fill this gap by insuring proper stress behavior near the wall
- Task:

  Design model to give correct mean profile, statistics at base of boundary layer
- Formulation:

  An additional stress term, based on the flow velocity, a drag coefficient and a canopy density function, a, is introduced into the momentum equations:

$$\left(\frac{\partial u_i}{\partial t}\right)_{canopy} = -C_D a|u|u_i \qquad (7)$$

For instance, Brown et al. set the value of C_D to 0.1 empirically to match wind tunnel data of the flow over canopy.

Rewrite previous equation (z is normal to canopy) as:

$$\left(\frac{\partial \tau}{\partial z}\right)_{canopy} = -C_D a|u|u_i$$

or

$$\tau_{canopy} = -\int - C_D a|u|u_i + \text{constant} \tag{8}$$

where τ_{canopy} is referred as the **'canopy stress'** and the constant of integration is set such that τ_{canopy} =0 at the top of the canopy. The canopy density function a is the leaf area per unit volume and has units of m^{-1}. It is defined for heights below the top of the canopy ($z<h_c$). One possible expression is:

$$a = a_0 \cos^3\left[(\pi z / 2h_c)\right] \tag{9}$$

At or above the canopy a=0. The purpose of the cos function is to ensure a smooth transition near the top of the canopy to avoid numerical issues associated with a sharp cutoff when a is constant throughout the canopy and then zero outside. The canopy height h_c is set equal to the horizontal grid spacing that is considered to represent the scale of the near-surface eddies. When the grid resolution in all three directions is comparable, a better choice for h_c is to set it equal to 2.5Δz. The value of the constant a_0 can be set such that the total stress at the first computational level off the wall (z=Δz) is equal to the local wall stress ρu_τ^2 .

Hybrid LES-RANS methods for high Reynolds number flows

Rationale for LES

- For many flows, RANS not accurate enough
- One RANS model cannot do all flows

But LES is too expensive

- Further development needed for complex high Reynolds number flows
    - One possibility: use complex LES wall models
    - Second possibility: use a **hybrid LES/RANS method**

Hybrid Method:

- Should have advantage of both
- Accuracy of LES, speed of RANS
- Should be able to treat
    - Complex flows
    - High Reynolds number flows

- Decide what types of flow to treat

  What are important properties?

  Which parts of flow should be simulated?

- Must ask whether all flows can be simulated with that particular method

Example: DES can simulate well massively separated flows around bluff bodies, but it is not very good at predicting much simpler flows compared even to RANS (e.g., flow in a channel)

Characteristics of target flows

- Flows dominated by large coherent structures, strong inherent unsteadiness
- Time scale of oscillation >> time scale of turbulence
- Treat only large scale part; model small scales

Possible target flows for these hybrid methods:

- Flows with characteristics given above
- Some examples currently being tackled:

  Bluff body flows

  IC engines

  Turbo machinery

  Flow around hydraulic structures (bridge piers, etc)

For method to be practical, we need

- Careful development, especially in modeling
- Careful validation; Need to assure that database is available
- Most successful example: **Detached eddy simulation** (described in next section)

Caution

- May not be applicable to all flows
- May be able to adapt RANS models
- RANS models used as part of a hybrid method may require different parameters
  This is because in this case we are modeling less of turbulence

Detached Eddy Simulation (DES)

- RANS as good as LES for boundary layer
- LES does better job in free shear flows

Idea:

- **Use RANS in boundary layer**
- **Use LES in detached region**

- Most difficult item for DES: getting transition right

- Original version starting from the Spalart-Almaras RANS model

- New versions starting from other RANS models (e.g., k-ω, SST, Streelets, 2001) are becoming available

Background and motivation for DES

DES is a hybrid RANS/LES non-zonal technique that can be applied at high Reynolds numbers as can RANS methods, but also resolves time-dependent, three-dimensional turbulent motions as in LES. Far from the boundaries the momentum transfer is dominated by large 'detached' unsteady eddies which are typically geometry dependent and could be resolved by LES without the vast increases in grid resolution necessary in LES of attached boundary layers which should normally resolve the coherent structures, including the wall streaks associated with these layers. Consequently, in this region most of the Reynolds stresses will be calculated directly and the resolved part of the total stress will be anisotropic. Especially for massively separated flows much of the burden of predicting the long-term Reynolds stresses is shifted from the turbulence model to the explicit averaging of a time-dependent 'vortex shedding' solution.

DES is somewhat similar to models which blend simple buffer-layer models (that use, for example, mixing-length approximations and damping functions near the walls) with subgrid-scale (SGS) models (generally, Smagorinsky based). One example is the model of Meng and Moin described above. In this sense, one can think of DES as primarily a wall model for high Reynolds number LES simulations, with the SGS model given by the particular formulation of DES away from solid boundaries and with the RANS model of DES giving the wall model. However, it differs from the more classical wall-layer modeling approaches for LES in the sense that DES can treat the entire 'RANS region' using much more advanced turbulence models (recall that even in the Meng and Moin's model a mixing length model was used in the RANS-TBL region). This is of relevance especially for massively separated flows, where the use of mixing-length models, and even algebraic models, does not yield very accurate predictions.

In DES there is a single solution field, and the transition between the RANS and LES regions, all coupled by the Navier-Stokes equations, is seamless in an application sense, i.e., without artificial transitions between the solution domains.

Spalart Almaras based DES

The S-A based DES model is based on a modification of the length scale in the destruction term of the one-equation eddy viscosity model developed by Spalart and Allmaras (1994). DES reduces to a RANS closure in the attached boundary layers (using the S-A model) and to a Smagorinsky-like subgrid scale model away from the wall (Spalart, 2000).

In the S-A RANS model, a transport equation is used to compute a working variable, $\tilde{\nu}$, used to form the turbulent eddy viscosity, ν_t. The DES formulation is obtained by replacing the distance to the nearest wall, d, by $\tilde{d}$ in the production/dissipation terms and model parameters. The distance $\tilde{d}$ is defined as,

$$\tilde{d} \equiv \min(d, C_{DES}\Delta) \qquad \Delta \equiv \max(\Delta x, \Delta y, \Delta z) \qquad (10)$$

where Δx, Δy, and Δz are the grid spacings. In ``natural" applications of DES, the wall-parallel grid spacings (e.g., streamwise and spanwise) are at least on the order of the boundary layer thickness and the S-A RANS model is retained throughout the boundary layer, i.e., $\tilde{d}=d$. Consequently, prediction of boundary layer separation is determined in the `RANS mode' of DES. Away from solid boundaries, the closure is a one-equation model for the modified SGS eddy viscosity:

$$\frac{D\tilde{\nu}}{Dt} = c_{b1}[1-f_{t2}]\tilde{S}\tilde{\nu} - \left[c_{w1}f_w - \frac{c_{b2}}{\kappa^2}f_{t2}\right]\left[\frac{\tilde{\nu}}{\tilde{d}}\right]^2 + \frac{1}{\sigma}\left[\nabla\cdot((\nu+\tilde{\nu})\nabla\tilde{\nu}) + c_{b2}(\nabla\tilde{\nu})^2\right] \quad (11)$$

Thus, DES switches to LES in regions where the grid spacing (in all directions) is smaller than the wall distance (to the nearest wall for complex geometries). Because in this latter region the more energetic turbulent eddies are generally not much smaller than the scale of the geometry, one may expect that the grid refinement necessary to obtain a much better flow description will not be too exaggerated compared to RANS. There is also a case to be made that far from walls DES acts as a dynamic LES model, as there is a transport equation for the 'equivalent' Smagorinsky constant which makes it vary from point to point and in time.

The eddy viscosity ν_t is obtained from:

$$\nu_t = \tilde{\nu} f_{v1}, \qquad f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}, \qquad \chi = \frac{\tilde{\nu}}{\nu} \tag{12}$$

where ν is the molecular viscosity. The production term is expressed as:

$$\tilde{S} = S + \frac{\tilde{\nu}}{\kappa^2 \tilde{d}^2} f_{v2}, \qquad f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}} \tag{13}$$

where S is the magnitude of the vorticity. The function f_w is given by:

$$f_w = g \left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6} \right]^{1/6}, \qquad g = r + c_{w2}(r^6 - r), \qquad r = \frac{\tilde{\nu}}{\tilde{S} \kappa^2 \tilde{d}^2} \tag{14}$$

The function f_{t2} is defined as:

$$f_{t2} = c_{t3} \exp(-c_{t4} \chi^2) \tag{15}$$

The wall boundary condition is $\tilde{\nu} = 0$ and the constants are $c_{b1} = 0.1355$, $\sigma = 2/3$, $c_{b2} = 0.622$, $\kappa=0.41$, $c_{w1} = c_{b1}/\kappa^2 + (1+c_{b2})/\sigma$, $c_{w2}=0.3$, $c_{w3}=2$, $c_{v1}=7.1$, $c_{t3}=1.1$, and $c_{t4}=2.0$.

When the production and destruction terms of the model are balanced (the flow is near equilibrium):

$$c_{b1}[1-f_{t2}]\tilde{S}\tilde{\nu} \approx \left[c_{w1} f_w - \frac{c_{b2}}{\kappa^2} f_{t2}\right]\left[\frac{\tilde{\nu}}{\tilde{d}}\right]^2 \quad \text{or} \quad \tilde{\upsilon} \approx coef * \tilde{d}^2 \tilde{S} \qquad (16)$$

the length scale $\tilde{d} = C_{DES}\Delta$ in the LES region yields a Smagorinsky eddy viscosity $\tilde{\nu} \propto \tilde{S}\Delta^2$, which varies in both space and time. The solution of the transport equation for the eddy viscosity accounts for transport and history effects analogous to dynamic formulations. The length scale redefinition away from solid boundaries increases the magnitude of the destruction term in the S-A model, drawing down the eddy viscosity and allowing instabilities to develop. Analogous to classical LES, the role of Δ is to allow the energy cascade down to the grid size; roughly, it makes the pseudo-Kolmogorov length scale, based on the eddy viscosity, proportional to the grid spacing. The model constant C_{DES} is of the order of one (C_{DES}=0.65 based on calibration in homogeneous turbulence by Shur et al., 1999), and it should be set so as the spectrum at high frequencies does not exhibit short oscillations, nor the decrease in the spectrum at high frequencies is too steep, in which case relatively large eddies are not resolved. Additional discussion of the model and implementation details (including an SST based formulation of DES) can be found in Spalart (2000), Strelets (2001), and Constantinescu and Squires (2003).

RANS vs. URANS vs. LES vs. DNS

Observations:

Δ=0 limit of LES is DNS

$\Delta \rightarrow \infty$ limit of LES is not RANS but rather some kind of average over entire flow

Alternative approach:

- Replace spatial filtering in LES by **time filtering**
- Small eddies have short time scales so they will be eliminated by the filtering similar as in space filtering
- Not often used in LES
- Causality requires uni-directional filter (you can only use the solutions at previous time steps)
- More difficult to apply, lots of practical problems
- But the advantage is that $\Delta_t \rightarrow \infty$ limit is RANS

Have continuum of methods

- DNS→LES→RANS
- Methods intermediate between LES, RANS possible?
- Called very large eddy simulation (VLES)
- Also called unsteady RANS, coherent structure capturing
- Hybrid LES/RANS methods fall in the same category

As (time) filter width (Δ_t) increases:

- Energy in unresolved scales increases
- More of the turbulence is modeled
- If we use eddy viscosity model eddy viscosity must increase
- Length scale of unresolved turbulence increases too

Limit $\Delta_t \rightarrow \infty$ is RANS

- If RANS is average over all unsteadiness then:
  - Viscosity used in RANS is just sufficient to stabilize flow
  - Viscosity used in RANS should be an upper bound for LES viscosity

- Further increasing its value would smooth mean velocity and thus it would produce incorrect mean profile

- Result is a quasi-laminar flow in which all velocity fluctuations are averaged out
- No unsteadiness should remain
- Question: What is ‘unsteady RANS (URANS)?’

Remarks about URANS

- In steady RANS, all unsteadiness removed

Observed problem: Sometimes RANS computation does not converge

- Time-dependent solution found rather than steady one
- Mean of time dependent RANS sometimes more accurate than steady RANS
- Claim: unsteadiness represents large structures
- Unlikely temporal behavior is correct

For instance, in many flows simulated to date URANS produces periodic flow when actual flow is not periodic. However, mean flow closer to experimental data than steady flow results

Problems with LES on very coarse grids:

- Smagorinsky model for LES viscosity is $\nu_T = (C\Delta)^2 |\overline{S}|$

  where Δ=filter scale
- On the other hand, RANS viscosity is $\nu_T = (CL)^2 |S|$

  where L=integral length scale

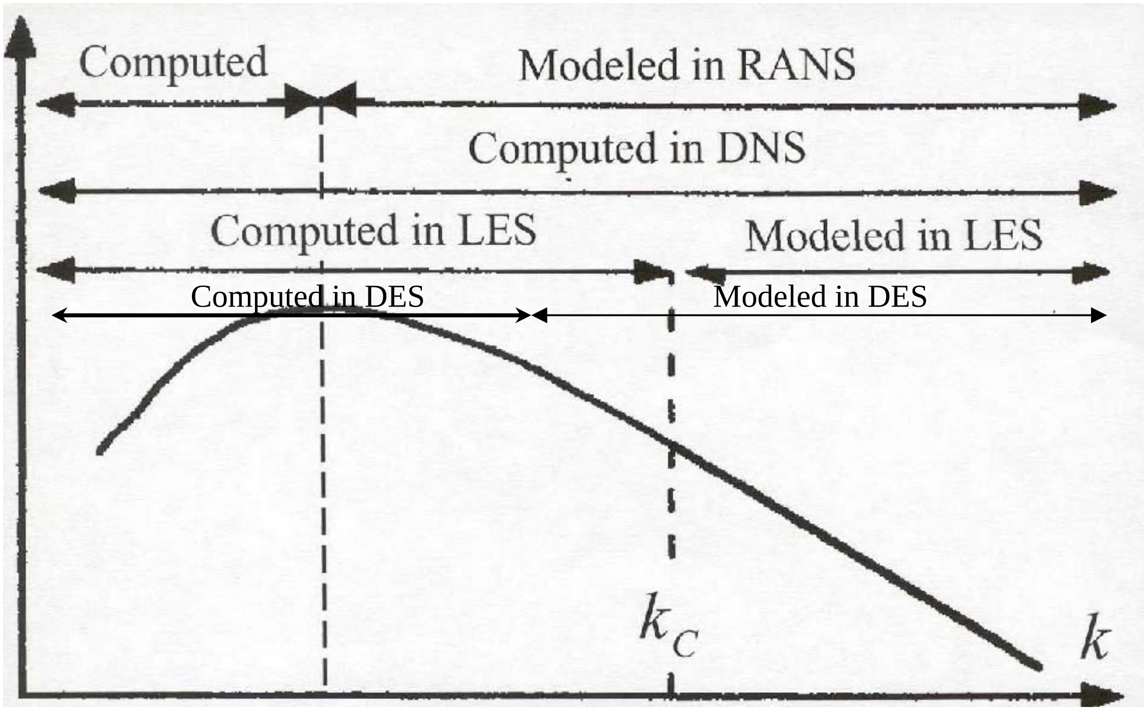
At high Re, it is possible that Δ>L

- Can and does happen in meteorology, oceanography
- If it does, LES has a problem

  Smagorinsky viscosity >RANS viscosity

  Correct model length scale should be smaller than Δ
- Need to predict model length scale
- May need another equation to determine it

- For example, two equation SGS model

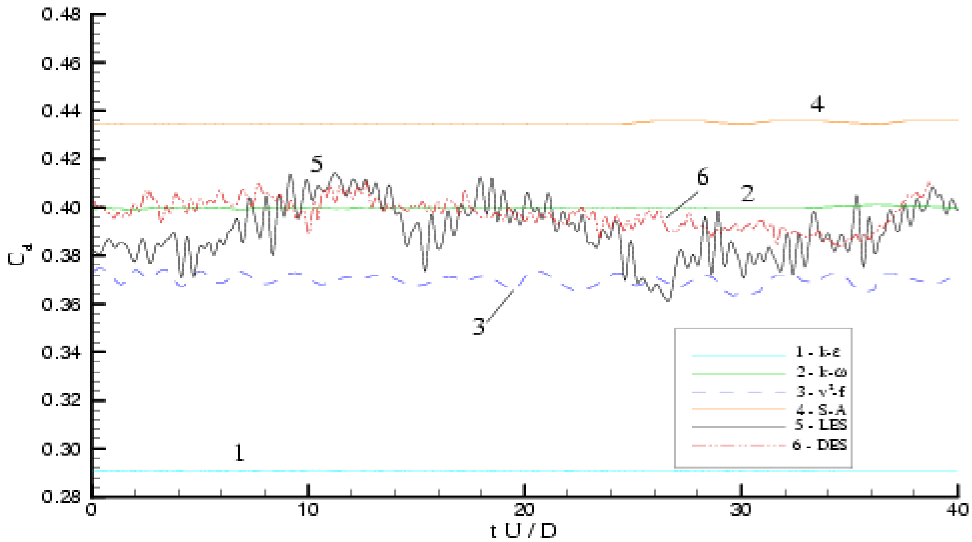


Sketch showing the turbulent energy spectrum plotted as a function of the wavenumber

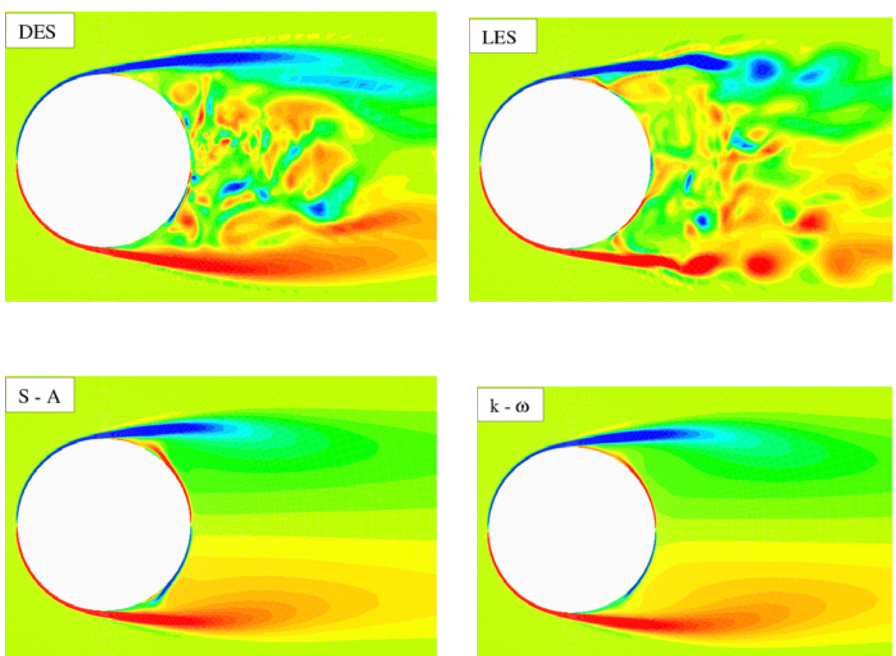
Discussion of RANS vs. LES vs. DES for massively separated flows (test case: flow over a sphere)

One can argue that DES should give very good predictions especially for massively separated flows (in particular for those with fixed separation from a sharp edge) because in these cases a rapid new instability in the detached shear layers dominates the turbulence inherited from the boundary layer. Thus, the fact that the shear layer calculated in DES has no eddies (RANS is used near the solid surfaces) should have no big influence on the capabilities of DES to resolve the large-scale eddies. For these flows, the solution has little sensitivity to the boundary-layer turbulence. On the other hand DES should resolve the most energetic eddies away from the wall. This is why DES appears to be better suited to capture the intricate vortex shedding mechanism, the wake structure, and is expected to predict more correctly the integral quantities of engineering interest compared with URANS methods at least for this category of flows.

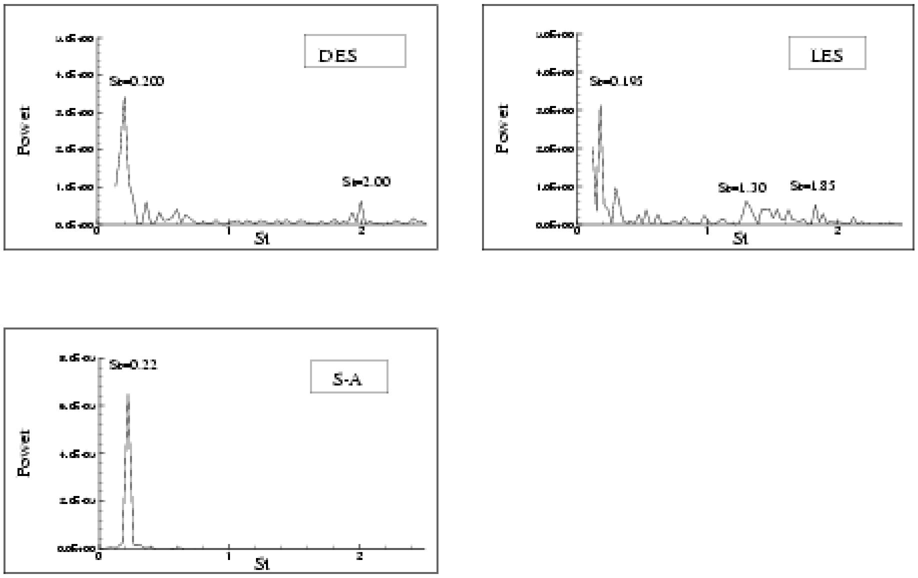
Recent efforts have been directed toward demonstrating the potential of DES in predicting complex separated flows compared to URANS. For instance Constantinescu and Squires (2003) in a study of the flow past a sphere showed that DES results for the subcritical flow past a sphere were close to full-domain LES (the dynamic Smagorinsky model was used) and were clearly superior to URANS especially when quantities describing the unsteadiness of the flow (such as the frequencies present in the wake, the time variation of the drag and lateral force coefficients) were compared. For instance, all URANS calculations (k-ε, k-ω, SA, v2-f) failed to detect the higher wake frequency associated with the instability of the shear layers and more importantly predicted very little energy (couple of order of magnitude lower in most cases) associated with the large scale shedding of hairpin-like vortices in the wake. In fact all solutions were practically steady (this is in contrast to the flow past a cylinder where URANS is generally more successful). In contrast, DES was able to accurately capture the large scale vortex shedding in the wake as well as the formation of vortex tubes in the separated shear layers. The success of DES sphere calculations over URANS could be explained based on the fact that the prediction of the long-term Reynolds stresses is dependent less on the turbulence model and more on the explicit averaging of a time-dependent 'vortex-shedding' solution. Moreover, the separation between the time scales of the unsteady deterministic motions and the one of the residual turbulence, which is implied inherently by any URANS model, is not present in measurements of velocity spectra.



Variation of the drag coefficient in time for the flow over a sphere, Re=10,000



Out-of-plane instantaneous vorticity in an azimuthal plane. Flow over a sphere.



Power spectrum for the time history of the drag. Flow over a sphere, Re=10,000

Compared to URANS, in a fully three-dimensional flow simulation DES is more expensive as typically the time steps are smaller and the grid in some regions of the flow has to be refined to capture the dynamically important eddies in that region (e.g., the vortex tubes in the detached shear layers), but the overall increase is generally less than one order of magnitude.

The main disadvantages of DES compared to LES is that additional empiricism is introduced near the wall compared to full-domain LES (but this makes the simulation feasible as far as the computational resources are concerned) and the fact that the subgrid model is constrained because of the calibration in RANS mode (we do not have a good understanding of the performance of the equivalent SGS model resulting from equations (11)-(13) as very little physics related to LES is incorporated in this model).