058:268 Turbulent Flows

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Handout: Turbulent Structures

Recall main properties of turbulence:

Three dimensional

- 2D turbulence exists, interesting mainly in geophysics

Unsteady

- Broad spectrum in time

Strong vorticity

- Strongest in coherent structures

Unpredictability

- Divergence of trajectories (inherent instability)

Broad spectrum of spatial scales

- Spectrum smooth, continuous

What are coherent structures?

- **Coherent structures** are features of the turbulence flow field. They are flow patterns that can be recognized atop the more disorderly motions. It associates a concrete form to the term '**eddy**'
- Structure and dynamics
    - Mainly due to mean strain
    - Different in each flow
    - Constantly evolving, being created and destroyed
    - Similar structures are present in transitional and homogeneous flows
- These structures are more **difficult to educe (to separate) in inhomogeneous flows**
    - Poor signal/noise ratio in high Reynolds turbulent flows

More on Structures and their importance to Turbulence?

- **Turbulence is partially deterministic,** that part is represented by the coherent structures. Though they contain roughly only **10% of the energy,** they are responsible for about **90% of the transport**
- The remaining **90% of the energy** in the flow is part of the **nondeterministic component of the turbulence (random noise)**. However, this nondeterministic component is thought to be due to the remains of previous coherent structures that are stretched, break and dissipate in time.

- **Coherent structures** are not identical (generally, they vary from flow to flow and also with the Reynolds number) or exactly repeatable in size, strength or time of occurrence. Rather they appear in many cases to be ‘quasi-periodical.
- **Eddies** in turbulence are responsible for mixing and the dissipative properties of the turbulent flow
- Statistical descriptions only give ensemble averages of random flow fields. Though ideas about **coherent structures** have not had a substantial impact on statistical turbulence modeling, they **have significantly contributed to the general (qualitative) understanding of turbulent shear flows** (free shear layers, boundary layers).
- The main motivation for studying coherent structures is to understand important properties (physics) of turbulent flows in terms of elementary structures. In many cases it can be insightful to look beyond the statistics of the flow to coherent structures (e.g., a helpful concept is the correlation length that can give information on the relative spacing and or size of the coherent structures).
- Advances in concepts of coherent structures have been successful in achieving engineering goals through the control of these structures by enhancing or destroying them. The following is a list of a few applications:

    - **Drag reduction**: The break up of long-lived, large coherent structures leads to drag reduction by shortening the range over which momentum is stirred.
    - **Heat Transfer enhancement**: Coherent structures have been modified to increase heat transfer in various applications.

- **Control of Aeroacoustic Noise**: Coherent turbulent structures that form in the jet potential core are associated with the aeroacoustic noise.
- **Mixing enhancement**: Fine-scale mixing of chemical reactants is enhanced when the large coherent structures break up into smaller eddies.

Importance of Transition for studying Coherent Structures

- The **structures** that occur **in fully turbulent** free shear **flows** are generally thought to be **similar to those seen in** the late stages of **transition** from laminar to turbulent flow
- In experiments, it is easier to visualize these structures in transitional flows compared to fully developed turbulent flows where small scales that can obscure the large structures are present.
- In transitional flows one can study the structures using linear theory (for initial stages) or nonlinear theory (especially for the later stages)
- Additional comments:
    - Homogeneous turbulence is another relatively easy case to study.
    - Even at high Reynolds numbers, where turbulence is very disorderly, one can sometimes identify coherent structures embedded in the irregular flow.

Large Eddies and Small Eddies

- **Large eddies** are associated with the energetic scales of motion. The term large eddy includes coherent vortices but also less clearly defined patterns of fluid movement.
- Most of the vorticity present in high Reynolds number turbulent flows is associated with the **small scales**. The small-scale vorticity is thought to be nearly isotropic in its structure. Small scales are close to isotropic (as eddies break successively into smaller eddies they lost their directional preference), so coherence is found at the large scales. This is also related to the usual concept of energy cascade. The concept of universality at small scales simply means that the smallest (typically close to and in the dissipative range) are nondescript.
- **Large eddies** do have directional preferences (they are geometry dependent) and generally we think of them as coherent structures.

Coherent Structures in Free Shear Flows

To better understand the processes that are responsible for the formation and evolution of the coherent structures in free-shear flows it is important to discuss the **stability** of these flows and the **transition** mechanism.

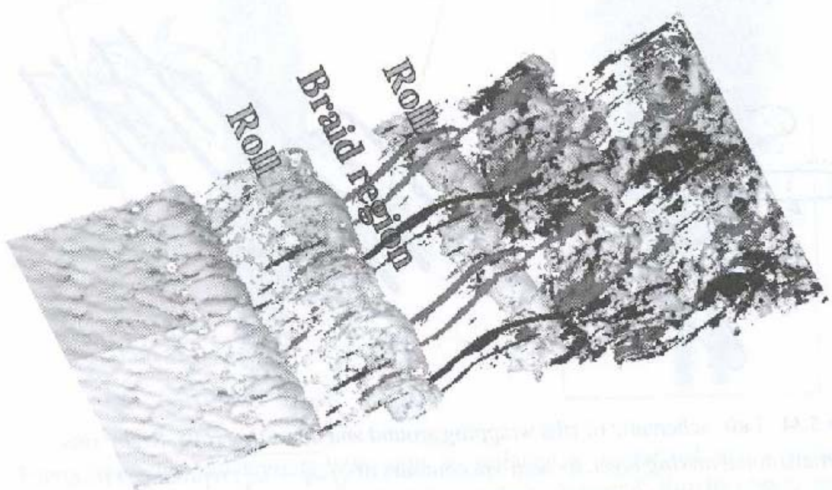

Figure 5.35 *Rolls and Ribs in a transitional mixing layer: contours of streamwise vorticity, ω_x. The grey and black streamwise contours, seen between the rolls, are the edges of positive and negative vortices of the braid region. Figure courtesy of P. Comte and P. Bégou (Comte et al., 1998).*

Instabilities in Free Shear Flows (Mixing layers, Jets, Wakes)

A. Mixing Layer

- Instability is Kelvin-Helmholtz

  - Inviscid mechanism (growth of instability is very fast)

- Velocity profile must have inflection point

  - Equivalent to maximum of vorticity

If a single mode dominates in the initial perturbation

- Laminar flow rolls-up into array of vortices

- Instability growth is eventually limited by nonlinearity when perturbation has ~10% of energy

- Final state has uniformly spaced vortices (obs: there is always some vorticity in the flow that is left outside of the vortices, this is also needed as we need to conserve linear momentum, etc.)

With broad-band initial perturbation

- Final state not as clean; Vortices not uniformly distributed

Results of instability theory

- Thin inviscid layer

  - Instability strongest at small scales

- Finite thickness layer

  - Preference for distance between vortices in final stages ~ 5 times the initial laminar thickness of the mixing layer

- Spanwise structures most unstable in all cases

Further results

- Viscous effects reduce growth rate at all scales

  - Effect larger at small scales so there is a finite k at which the growth rate has its maximum (for the case with viscosity)

- Slightly oblique modes are nearly as unstable and they can play an important role in the transition

Vortices in Rolled-up Layer:

- If noisy perturbations (natural transition) are present then

  - Spacing, size, strength of vortices may vary

  - Braids contain some vorticity

  - Stagnation line at center of braid

- Regular array produced only if:

  - Perturbation is regular (e.g., sinusoidal). This requires a very quiet wind tunnel

Recall properties of instability

- Preference for scales (vortex diameters) ~ 5 x laminar thickness
- Viscous effects reduce growth rate at small scales
- Spanwise structures favored; slightly oblique modes may be important

Growth of Rolled-Up Layer (2D)

After roll-up is complete

- Only perfectly regular array is stable
- Other arrangements unstable to many types of 2D and 3D perturbations
- The (2D and 3D) processes in the mixing layers due to the presence of these perturbations are discussed next.

Some 2D processes:

- If vortex separation is irregular:

  - **Pairing**: amalgamation of two vortices (the two vortices start rotating around each other, they deform and get closer to each other, more and more stretching is present, finally they merge due to the diffusion)

    * Produces array of larger vortices (still 2D)

    * Vortex diameter and spacing approximately double original values, requires entrainment of external (maybe non-turbulent) fluid

  - Merging of three vortices is also possible

    * Requires special perturbation (typically this kind of perturbation does not occur naturally)

- If vortex size is irregular

  - **Tearing** is possible (this is an important process as it occurs naturally)

    * One starts with a small vortex between two large ones. The two large vortices stretch, divide and finally absorb the smaller one

  - Other processes possible

Observation: **All these processes maintain two dimensionality**

Paths to three-dimensionality: vortex array is unstable to 3D perturbations

- Several modes known

1) Corcos-Lin instability

- Vorticity, strain exist in braid; combination unstable to wavy perturbation

When a small spanwise vortex is situated between two larger spanwise vortices, the vortex line corresponding to the smaller vortex gets disturbed and starts being stretched due to the strain present in the flow. Stretching amplifies vorticity leading to streamwise vorticity. With time some regions of the vortex line are drawn toward the two main vortices and connects with them. In the end we get an array of streamwise oriented vortices of alternating signs. This is the mechanism responsible for the development of the ribs in mixing layers and in the wake behind a cylinder.

2) **Widnall instability**

- Usually for vortex rings (e.g., in round jets) but process is similar for straight vortices
- Can occur when the vortex develops a kink or a series of kinks (due to an instability wave)
- The self-induced motion lifts and stretches the vortex leading to streamwise vorticity
- In free shear flows local Widnall instabilities are quite often encountered. They force transition to 3D.

3) Local or helical pairing

- Similar to 2D pairing but

  - Only small parts of neighbor vortices merge

  - Other parts merge with other vortices

  - Leads to complex pattern

  For instance, if we have three vortex lines oriented in the spanwise direction and all vortices have vorticity of same sign then, if disturbances are present, the first two vortices will connect and merge in some places while in other places the last two vortices will merge together. Generally, in regions where pairing occurs there is local transfer of energy from small scales toward the larger ones, while in the remaining fluid most of the energy transfer is from the large scales toward the small ones as the flow becomes turbulent

4) Vortex reconnection (not very important)

- Ring vortices can merge and then split again
- Final state different from initial state
- Leads to more complex pattern

  For instance, given two initial ring vortices one can tag the fluid elements and vorticity in one of them as red and in the other one as blue. After merging and re-splitting one can end with two identical vortices to the initial ones, but each of those vortices is formed from equal parts of blue and red vorticity. One can numerically simulate this process.

5) **Other processes:** Combinations, other to be discovered?

Later stages of flow

- Vorticity distribution is much more complex

  - Merging produces larger vortices

  - Stretching reduces diameter of vortex and sends energy to the smaller scales

B. Jets and Wakes

Slightly more complicated free shear flows

- Processes are similar to ones encountered in mixing layers
- Laminar flow contains velocity maximum & two vorticity maxima, one of each sign
- For plane jets and wakes the two sides of the jet/wake are subject to instabilities like in a mixing layer
- Each side rolls up separately, similar to the processes present in a mixing layer
- The end result: vortices of different signs from each side
- Staggered arrangement favored (is more stable): von Kármán street

Later stages of transition

- Not as well studied as for mixing layers, flow more complex than mixing layer
- Processes are thought to be similar to the ones present in mixing layers
- Each side is behaving as a mixing layer but interaction between the sides is important
- Vortices stretched from one side to other

Coherent structures in mixing layers

Mixing layer coherent eddies consist of large **rolls** of spanwise vorticity, with streamwise **ribs** superimposed. As already mentioned, the spanwise rolls correspond to the vortices developed due to the two-dimensional Kelvin-Helmholtz instability. The gap between the rolls is called the **braid region**. The dominating structures in the braid region are **ribs** of streamwise vorticity, which are the legs of vortex loops than span the gap between successive rolls, as in figure 5.35. Ribs contribute significantly to the entrainment of fluid into the mixing layer. The ribs are stretched by the spanwise rolls, while at the same time they distort the surface of the spanwise rolls. In fully turbulent flows, the braid region contains a good deal of small scale turbulence.

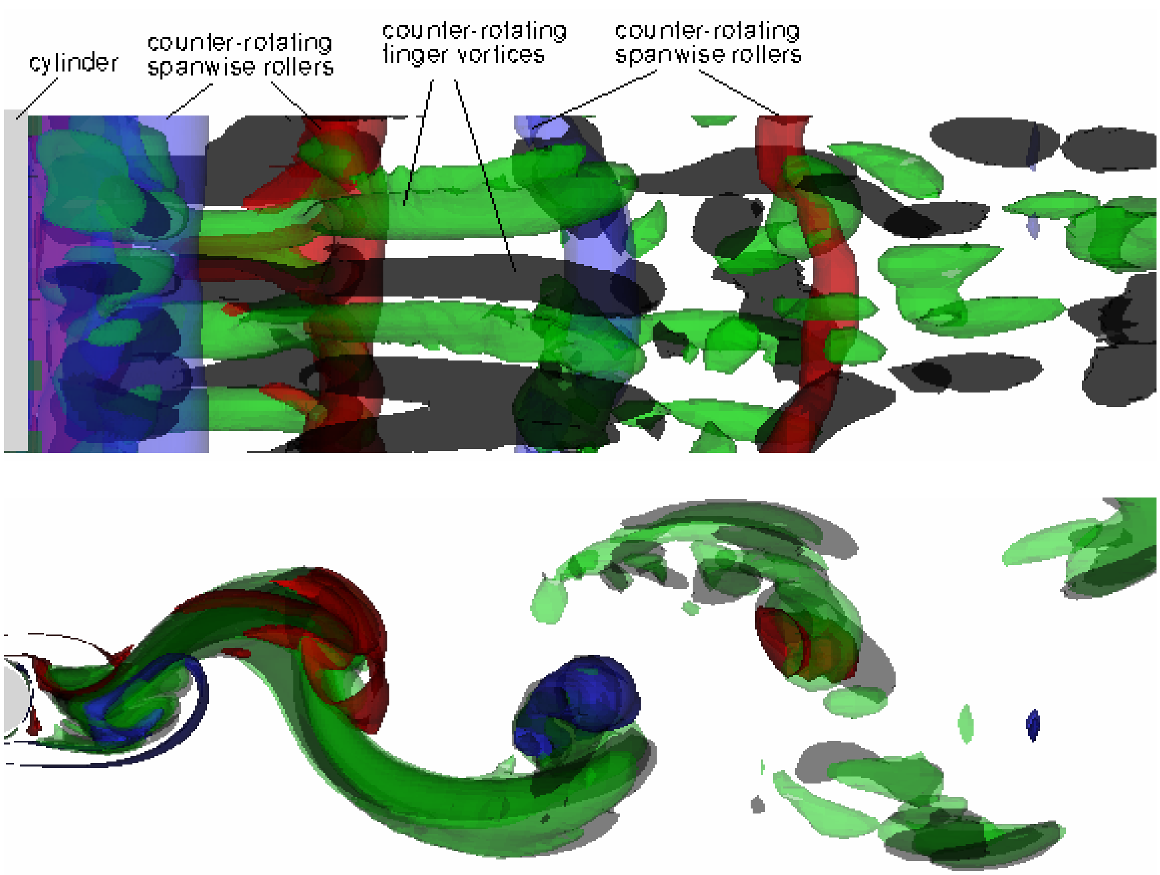

Fig. LES simulation of the turbulent wake of a cylinder at Re=3,900 (Constantinescu et al., 2002)

Double Rollers (fig 5.36) are present in planar jets, wakes and mixing layers. They are vortices aligned at approximately 40^0 to the shear, irrespective of its direction. Fig. 5.36 shows their alignment in a mixing layer. In a plane jet, two sets of rollers are inferred, forming a horizontal 'V' shape, such that each set is aligned to the shear in the upper and lower parts of the flow. While the spanwise rolls are responsible for the mixing layer growth in the transitional stage, they are probably not the dominant coherent structures of fully turbulent mixing layers. Streamwise, sloping entrainment eddies as in Fig. 5.36 are probably more important to the overall turbulent mixing-layer dynamics. However, external forcing can be used to stimulate the spanwise rolls, as illustrated in Fig. 5.37.

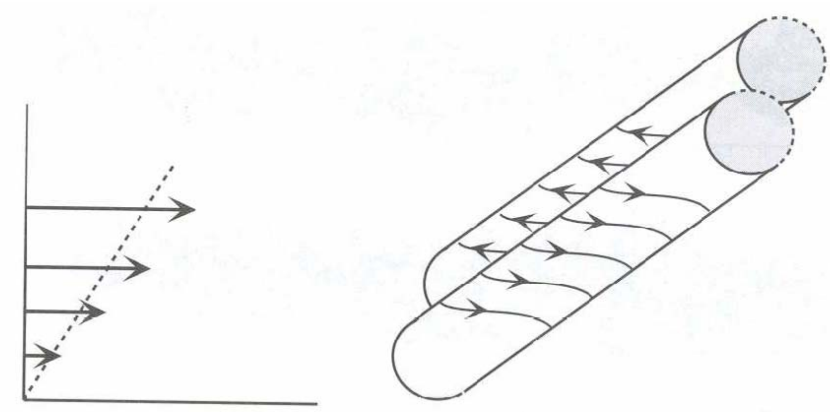

Figure 5.36 *Double rollers in free-shear flows, after Townsend (1976).*

Fig. 5.36 **Double Rollers** from Durbin & Pettersson Reif (2001) *Statistical Theory and Model for Turbulent Flows*

The upper panel of Fig. 5.37 is a natural fully turbulent mixing layer that shows little evidence of rolls. In contrast in the lower panels where periodic forcing of suitable frequency was applied, spanwise structures (rolls) are forming with the effect of increasing the growth rate of the mixing layer by the mechanism of vortex pairing. This is observed well in the middle of the lower panel where two structures are seen in the act of pairing. As time evolves they roll around each other and merge into a larger eddy. In general, periodic forcing can create coherent structures in both mixing layers and jets. Forcing is one possible way of controlling shear layer turbulence.

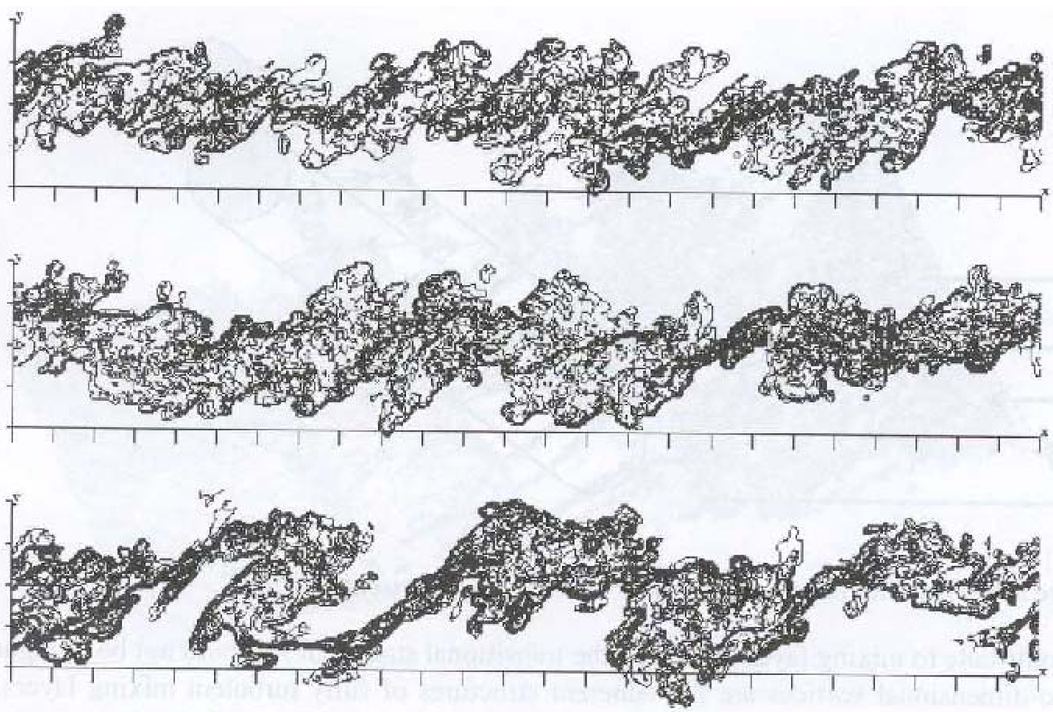

Figure 5.37 *From Rogers & Moser (1994); reproduced with permission. Contours of a passive tracer delineate mixing layer eddies. Top: a natural mixing layer; middle: a mixing layer subject to moderate, periodic forcing; lower: a mixing layer with strong forcing.*

Fig 5.37 **Forced Mixing Layer** from Durbin & Pettersson Reif (2001) *Statistical Theory and Modeling for Turbulent Flows*

Another engineering significance of turbulent coherent eddies is that they entrain free-stream fluid into a growing shear layer. The engulfed free stream fluid then acquires vortical turbulence as it is mixed by smaller scale eddies.

Coherent structures in jets and axisymmetric wakes

In a **round turbulent jet** in its early stages (before transition to turbulence has taken place), the main coherent structures are **vortex rings**. They are the equivalent of the spanwise rolls present in mixing layers. These rings form because the circular nozzle of the jet imposes a condition of axisymmetry on the large-scale structures. This has lead to the suggestion that the axisymmetry of the vortical rings can be broken by corrugating the nozzle lip. The corrugations increase the break-up of axisymmetric vortices into smaller, irregular eddies. This is of interest because these vortical rings are often harmful to engineering goals. Small-scale mixing can be promoted by encouraging the breakup of larger structures.

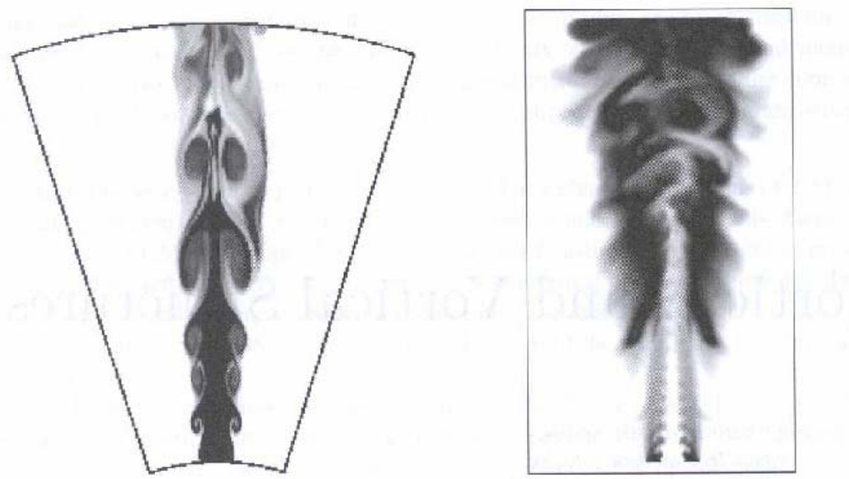

Figure 5.33 *Jets undergoing transition to turbulence, visualized via a scalar concentration field. From numerical simulations by Steiner & Busche (1998) and Danaila & Boersma (1998).*

While it might be reasonable to think that **spheres** would shed similar vortical structures to round jets (vortex rings), they do not. Even if the mean flow is axisymmetric, the sphere does not shed vortex rings.

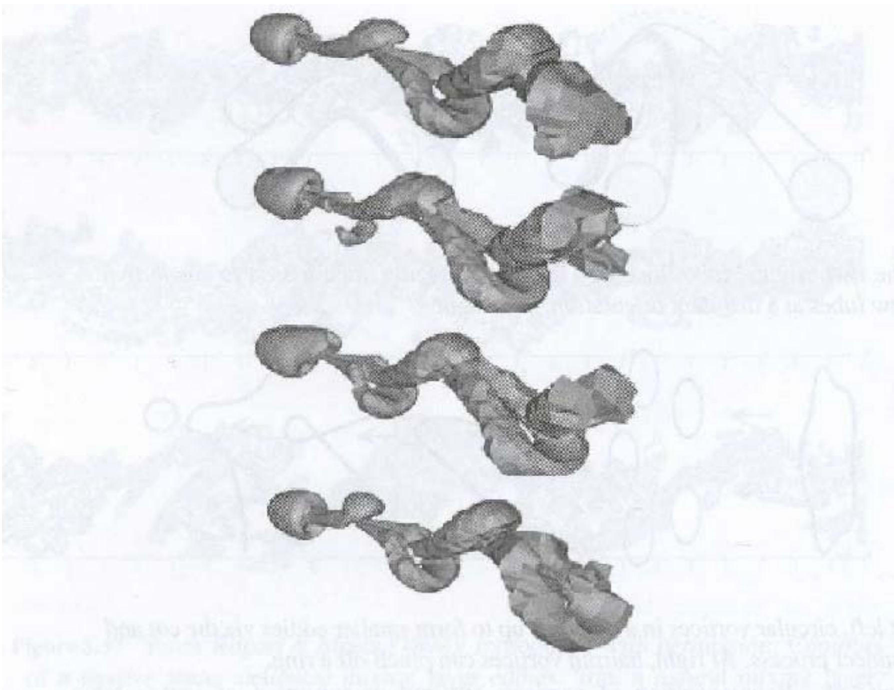

Figure 5.39 *Vortical structures downstream of a sphere. The surfaces enclose regions where rate of rotation is greater than rate of strain. The loops rotate direction to create a helical signature. From Johnson & Patel (1999), reproduced with permission.*

The wake of the sphere has a **helical structure** (at least when the flow at separation is laminar with transition taking place in the detached shear layers) formed by **successive shedding of hairpin like vortices at random azimuthal angles.** The picture in Figure 5.39 corresponds to a laminar wake in which the hairpin-like vortical structures that are shed in the wake are similar to those shed at higher Reynolds numbers (turbulent wakes) but in which the shedding is locked in a particular azimuthal plane.

Coherent Structures in Shear Flows

- Most important type of engineering flow
- Important structures are vortices
  - Spanwise and streamwise
  - Hairpins
- Most important sub-case: boundary layers

Background: Widnall instability in shear flows

- Mean shear enhances stretching
- Induces kinks in neighboring vortices (let's say parallel spanwise vortex lines)
- In shear flows, fluid farther from the wall is faster so the deformation is elongated even more
- Eventually a hairpin-like vortex develops locally out of the original straight spanwise oriented vortex line. The hairpin structure is oriented at about 45% relative to the mean streamwise direction

Coherent Structures in Boundary Layers

These are three main types of widely accepted coherent structures in boundary layers:

- **Low Speed Streaks**: $y^+ < 40$ (near the wall)
- **Hairpin or Horseshoe Vortices and Rolls**: $y^+ < 100$ (central portion of b.l.)
- **Viscous Superlayer**: $y \approx \delta$ (undular interface with the free-stream at the edge of the boundary layer)

Low Speed Streaks

- Correspond to relatively slow moving fluid with streamwise velocity about half of the local mean.
- Are spaced (all dimensions are in wall units) $\Delta z^+ \approx 100$ apart in the spanwise direction z, have a width of $\Delta z^+ \approx 30$ and can have a length of $\Delta x^+ \approx 2000$.
- Low speed streaks occur between the legs of the hairpin vortices, where flow is displaced upward from the surfaces so that it convects low momentum fluid away from the wall.
- The streaks have a characteristic behavior, known as **bursting**. With increasing downstream distance, a streak migrates slowly away from the wall, but then, at some point (typically around $y^+ \approx 10$), it turns and moves away more quickly, a process referred to as streak lifting or **ejection**. As the streaks are lifted they become unstable and are broken down into finer-scale motions (so bursting refers at the whole process described and includes the ejection). These streaks have been described as having a jet like character.

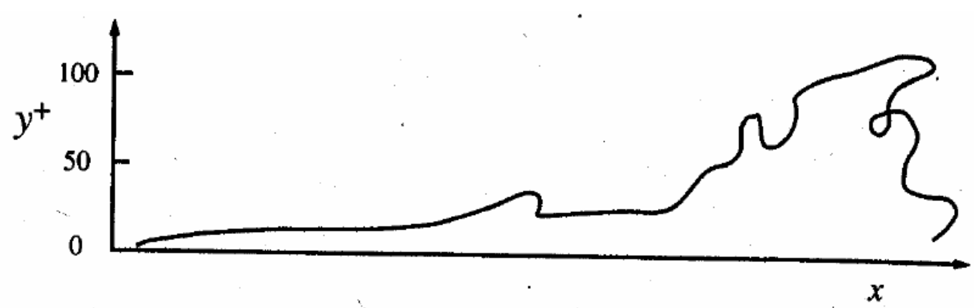

Fig. 7.40. A dye streak in a turbulent boundary layer showing the ejection of low-speed near-wall fluid. (From the experiment of Kline *et al.* (1967).)

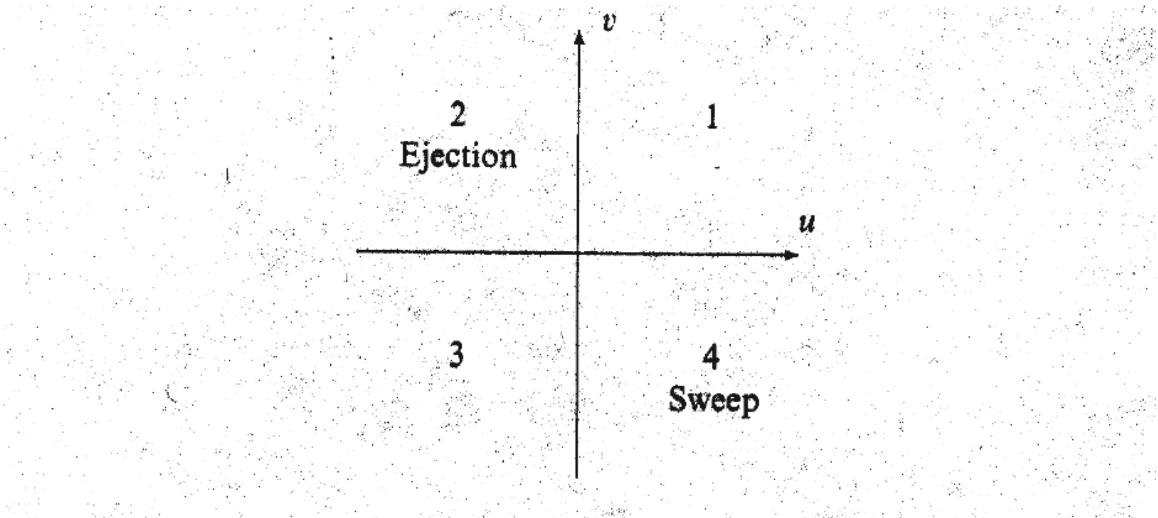

Fig. 7.41. The u–v sample space showing the numbering of the four quadrants, and the quadrants corresponding to ejections and sweeps.

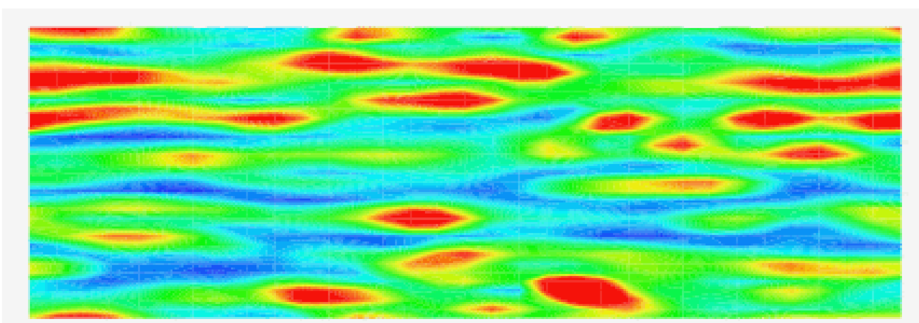

Fig. Low speed Streaks (blue color) in a channel flow (plane parallel to the wall situated at about 10 wall units)

- On the outside of the pair of legs of the hairpin vortices, high speed streaks are observed (red color in above figure). They are produced by convection of high momentum fluid near the wall and are generally much shorter than the low speed streaks.
- Ejection of streaks requires, due to continuity equation, a flow toward the wall in some other regions. These events are called **sweeps**. So, in summary in an

ejection u<0 and v>0, while in a sweep u>0, v<0. Thus, both ejections and sweeps require that <uv> <0 (where < > denotes Reynolds averaging). These events therefore lead to positive turbulent kinetic energy production (as the main term in P is $-<uv>\frac{\partial\langle U\rangle}{\partial y}$ and $\frac{\partial\langle U\rangle}{\partial y}>0$ with y pointing away from the wall toward the flow) so they produce turbulent energy. One should point out that production is on average positive without ejections and sweeps, so production is not exclusively caused by these turbulent structures.

Hairpin of Horseshoe Vortices and Rolls

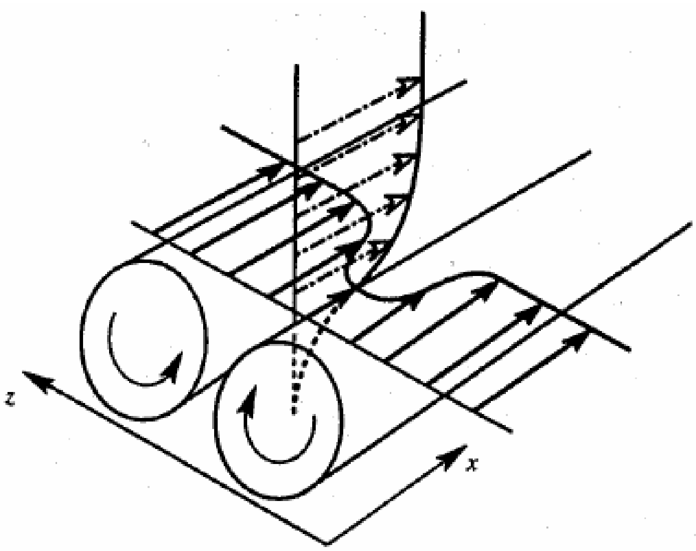

Fig. 7.42. A sketch of counter-rotating rolls in the near-wall region. (From Holmes *et al.* (1996).)

Rolls are pairs of counter-rotating streamwise vortices that are the dominant vortical structures in the near-wall region defined by $y^+ < 100$ (they are associated with the initial part of the legs of the hairpin vortices). These rolls account for the existence of streaks since close to the wall, between the rolls, there is a convergence of the flow in the plane of the wall. The fluid moving away from the wall between the rolls has a reduced axial velocity, which leads to the velocity profile sketched in Figure (7.42). These profiles contain inflexion points and so they are inviscidly unstable, and are associated with streak lifting or bursting.

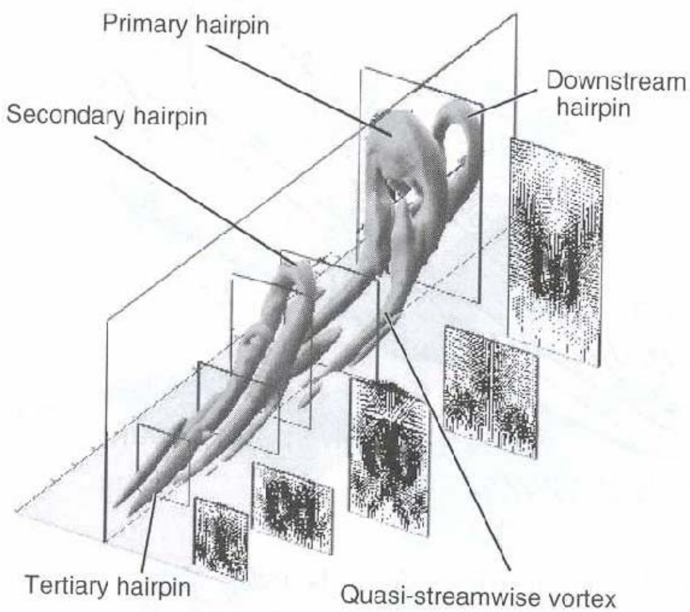

Figure 5.41 *Hairpin eddies in low Reynolds number channel flow. Courtesy of R. Adrian, J. Zhou & S. Balachandar, reproduced with permission.*

Hairpin or **Horseshoe Vortices** are the dominating flow features further away from the wall in the boundary layer. These structures are sometimes described as double cones since the top loop is tough to visualize. They are aligned at about 45^0 from the wall and nearly attached to the wall surface. One should point out that the eigenvectors of the rate of strain tensor in a parallel shear layer flow are vectors at $\pm 45^0$. The 45^0 orientation of the attached eddies tends to align them with the principal directions of the strain. This suggests that these structures extract energy from the mean flow in an efficient way.

The low Reynolds number simulation in Figure 5.41 suggests a process by which self-generation of vortex loops might lead to a packet of hairpin vortices. As the vortex loops are lifted from the surface they are stretched by the mean shear, ultimately becoming hairpins, with long, nearly straight legs, lying at about 45^0 to the flow. This stretching causes them to be very elongated in high Reynolds number flows. The legs of the primary hairpin vortex induce a distortion in the underlying boundary layer that locally intensifies the spanwise vorticity. The local patch of high vorticity becomes kinked into another horseshoe vortex, which lifts from the surface. One can link this scenario to the above mentioned burst and sweep events. The lift up is associated with the burst while the regeneration is being initiated by the sweep. In a channel flow the length of a hairpin vortex is of the order of the channel half width, while their cross-stream dimension (width) scales with the viscous lengthscale $\delta_\nu=\nu/u_\tau$.

Viscous Superlayer

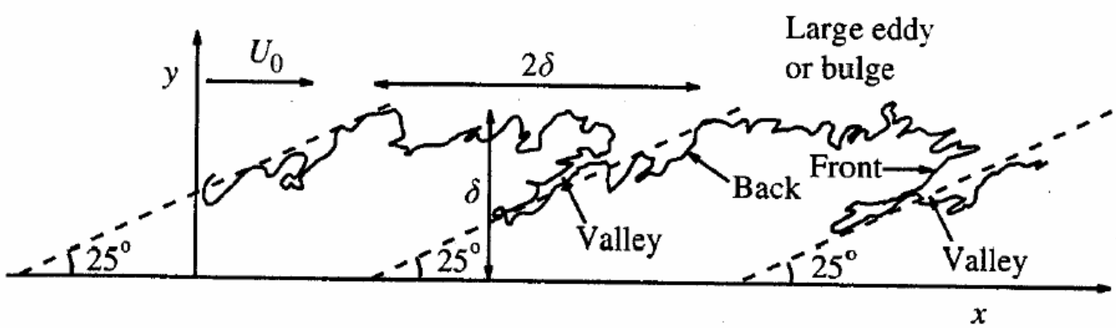

Fig. 7.44. The large-scale features of a turbulent boundary layer at $Re_\theta \approx 4{,}000$. The irregular line – approximating the viscous superlayer – is the boundary between smoke-filled turbulent fluid and clear free-stream fluid. (From the experiment of Falco 1977.)

At the upper edge of the boundary layer, where the flow is intermittent, the dominant turbulent structure is the **Viscous Superlayer** separating the turbulent boundary layer fluid from the irrotational free-stream fluid. This structure consists of large eddies, or bulges, approximately centered at 0.8δ (δ is the mean width of the boundary layer at a given streamwise location), of length δ to 3δ, and about half as wide in the spanwise direction, which are separated by deep valleys of non-turbulent fluid which are inclined at an angle of 20-25^0 from the wall. Their mean convection velocity is smaller (0.85-0.9U_0) that the local free-stream velocity U_o. These structures have properties which scale with U_o and δ. It has been proposed that these large eddies are produced by the envelope of successive (patches of) hairpin vortices. Entrainment occurs via engulfing into the bulges, in conjunction with mixing by small eddies. Deep intrusion of laminar fluid between the turbulent zones is also observed.